Energy Detection Under Interference Power Uncertainty

Jingwen Tong, Ming Jin, Qinghua Guo, and Long Qu

***Abstract*— This letter concerns energy detection (ED) under interferences from surrounding users, where the challenge is that the distribution of the aggregate interference power has an unbounded support. With the assumption of log-normal large-scale fading of interferences and high interference-to-noise ratio, we derive an approximate but explicit expression for the decision threshold of ED that guarantees a given average false-alarm probability, and analyze its signal-to-interference ratio (SIR) wall. Simulation results verify the theoretical results and demonstrate that the proposed ED has a lower SIR wall and a better performance than the bounded worse behavior-based ED and the estimated background (interference and/or noise) power-based ED.**

***Index Terms*— Cognitive radio, energy detection, interference power, unbounded support.**

I. Introduction

COGNITIVE radio (CR), which allows the operation of secondary users (SUs) in the occasionally unused spectrum of primary users, has been recognized as a promising technology for alleviating the problem of wireless spectrum resource shortage. To probe available spectrum holes and avoid intra- and inter-system interferences among unlicensed/licensed systems, one of the most important functionalities of cognitive radio is spectrum sensing [1].

Energy detection (ED) is one of the most popular techniques in spectrum sensing due to its low computational complexity and semi-blind property (it requires no prior knowledge of primary signals) [1]. Given an exact background (interference and/or noise) power, ED would deliver a superior performance [2]. However, the exact background power is often unavailable in practice, and background power uncertainty may cause severe performance degradation [3].

In the literature, the noise (power) uncertainty, together with the signal-to-noise ratio (SNR) wall (the minimum requirement of SNR for achieving a reliable detection performance), has been analyzed thoroughly in [4] and [5]. A lot of efforts have been devoted to computing the decision threshold of ED under different noise uncertainty models [2]–[7]. The existing noise uncertainty models fall into two categories. One is referred to as bounded worse behavior (BWB) model. In this model, the exact noise power is assumed to be unknown but falls into a bounded range, and the upper bound is employed to replace the exact noise power in determining the decision threshold [3], [4], [6], [7]. BWB model guarantees that false-alarm probabilities do not exceed a specified value (e.g., 10% in IEEE 802.22). However, the replacement of the noise power with its upper bound increases decision thresholds and thereby decreases detection probabilities. The other model is referred to as estimated background power (EBP) model, where an estimated noise power (ENP) is used [2], [4], [5]. In [2], a noise power estimator was proposed that makes ED robust against noise uncertainty. In [4] and [5], the log-normal and Gaussian distribution assumptions for ENP were used in analyzing the SNR wall.

To the best of our knowledge, the impact of interferences from multiple users on ED has not been well investigated. In practice, multi-user interferences are often present in many scenarios, such as LTE-A, WiFi and WiMAX networks in which users simultaneously operate in the same frequency band. The difficulties in handling these interferences are twofold. One is that the support of the distribution of interference power may be unbounded. Hence, BWB model would not be applicable. The other is that the interference power may vary significantly from one sensing interval to another one. Hence, the estimated interference power from previous sensing intervals would deviate from the one at current sensing interval. In this work, we analyze the decision threshold of ED from the average false-alarm probability, with the assumptions of log-normal large-scale fading of interferences and high interference-to-noise ratio (INR). An approximate but explicit expression for the decision threshold of ED is derived and the signal-to-interference ratio (SIR) wall is analyzed. Simulation results show that the proposed method achieves a lower SIR wall and a better performance than the BWB- and EBP-based EDs.

II. Signal Model and Problem Formulation

Consider a scenario shown in Fig. 1 where a secondary (cognitive) user and a primary network with several users share the same spectrum resources. As shown in Fig. 1, we assume that one of the primary network users is in the coverage of the secondary user, while others users of the primary network are not. The signal from the user within the coverage of the secondary user is regarded as primary signal by the secondary user, and the user may be potentially interfered by the secondary user. The signals from other users are treated as interferences by the secondary user, and the transmission of the secondary user will not affect these users. To avoid interfering the user within its coverage, the secondary user needs to perform spectrum sensing under the possible interferences

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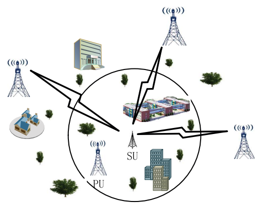

Fig. 1. Scenario of cognitive radio with interferences from surrounding users.

from the primary network users outside the coverage of the secondary user.

A. Signal Model

Let $\mathcal{H}_0$ and $\mathcal{H}_1$ be the hypotheses representing respectively the absence and presence of primary signals. Under this binary hypothesis, the received discrete-time signal $x(n)$, $n = 0, \cdots, N-1$ with N being the total number of samples, at the secondary user is given by

$$x(n) = \begin{cases} w(n), & \mathcal{H}_0 \\ s(n) + w(n), & \mathcal{H}_1 \end{cases} \tag{1}$$

where $s(n)$ denotes the primary signal, and $w(n)$ denotes interferences plus noise. We assume that $s(n)$ and $w(n)$ are independent of each other. In addition, the samples of $s(n)$ are independent with mean zero and variance σ_s^2, and the samples of $w(n)$ are also independent with mean zero and power I. In this work, we investigate the case that $w(n)$ is dominated by the interferences, i.e., noise power is negligible compared with the aggregate interference power [8]. This often occurs when secondary users try accessing the spectrum licensed to networks with high SNRs [8]. It is assumed that the users in the primary network have power/contention control, and the interferences experience independent large-scale fading and small-scale fading. As the interference power with large-scale fading usually follows the log-normal distribution, the aggregate interference power can be regarded as a sum of randomly weighted log-normal variables, which can also be modelled as a log-normal distribution [9], [10], i.e., $I \sim Log\mathcal{N}(\mu, \nu)$ where μ and ν denote the mean and variance of the natural logarithm of I [9], [11]. The probability density function (PDF) of I is given by [5]

$$f(I) = \frac{1}{I\sqrt{2\pi\nu}} e^{-\frac{(\ln I - \mu)^2}{2\nu}}, \quad 0 < I < \infty \tag{2}$$

where $\ln(\cdot)$ denotes the natural logarithmic function. Although the interference power at current sensing interval may be unknown, its statistical parameters, i.e., μ and ν, can be estimated by using multiple previous sensing intervals with relatively small signal power in periodic sensing [12], [13] or using signals from neighbouring frequency bins in wideband sensing [14].

Let the mean and variance of I be σ^2 and $(\rho - 1)\sigma^4$, respectively. It is not hard to show that $\rho = e^{\nu}$ and $\sigma^2 = e^{\mu + \nu/2}$. With the same definition as in [5], the uncertainty of the interference power I is defined as ρ, and the uncertainty in decibels is given by $U_{\text{dB}} = 10 \log_{10} \rho = 10\nu \log_{10} e$, which is proportional to ν. In addition, we define the SIR as

$$\text{SIR} = \frac{\sigma_s^2}{\mathbf{E}[I]} = \frac{\sigma_s^2}{\sigma^2} \tag{3}$$

where $\mathbf{E}[\cdot]$ denotes the expectation operator.

B. Problem Formulation

The test-statistic of the ED is given by

$$T_{\text{ED}} = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2. \tag{4}$$

With a sufficiently large N and by the central limit theorem, T_{ED} approximately follows Gaussian distributions under $\mathcal{H}_0$ and $\mathcal{H}_1$ as

$$T_{\text{ED}}|\mathcal{H}_0 \sim \mathcal{N}\left(I, \frac{1}{N} I^2\right) \tag{5}$$

and

$$T_{\text{ED}}|\mathcal{H}_1 \sim \mathcal{N}\left(\sigma_s^2 + I, \frac{1}{N}\left(\sigma_s^2 + I\right)^2\right), \tag{6}$$

respectively. Conditioned on I, the false-alarm probability and detection probability are given by

$$P_f(\lambda|I) = Q\left(\frac{\lambda - I}{\sqrt{N^{-1}}I}\right) \tag{7}$$

and

$$P_d(\lambda|I) = Q\left(\frac{\lambda - (\sigma_s^2 + I)}{\sqrt{N^{-1}}(\sigma_s^2 + I)}\right) \tag{8}$$

where λ denotes the decision threshold, and

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-\frac{u^2}{2}} du. \tag{9}$$

In the literature, e.g., [3], [6], and [7], the power I is assumed to be in a bounded range $[I_l, I_u]$. To guarantee that the false-alarm probability does not exceed a given one P_f^*, the ED in worst-case replaces I with the upper bound I_u in determining the decision threshold, which is given by

$$\lambda = I_u + \sqrt{N^{-1}} I_u Q^{-1}\left(P_f^*\right). \tag{10}$$

Under log-normal interferences, I has an unbounded support [9], [11], and the worst-case operation will severely degrade the performance of ED. To deal with this issue, we propose to employ the Bayesian criteria for computing the decision threshold and analyzing the SIR wall.

III. Decision Threshold and SIR Wall

A. Decision Threshold

The false-alarm probability in (7) is conditioned on the random variable I. Thus, we can obtain the average false-alarm probability as

$$\overline{P}_f(\lambda) = \mathbf{E}\left[P_f(\lambda|I)\right] = \int_0^{\infty} P_f(\lambda|I) f(I) dI \tag{11}$$

where $f(I)$ is given in (2). The decision threshold λ for a target false-alarm probability P_f^* can be obtained by solving the implicit equation

$$\int_0^{+\infty} P_f(\lambda|I) f(I) dI = P_f^*. \tag{12}$$

In the following, we will derive an approximate but explicit expression for the decision threshold.

Note that $P_f(\lambda|I)$ is a monotonic function of I. By applying the second mean value theorem to (11), there exists $\xi(\lambda) \in (0, +\infty)$ which is associated with λ, so that [15]

$$\begin{aligned}\overline{P}_f(\lambda) = &\lim_{I\to 0^+} P_f(\lambda|I) \int_0^{\xi(\lambda)} f(I) dI \\ &+ \lim_{I\to +\infty} P_f(\lambda|I) \int_{\xi(\lambda)}^{+\infty} f(I) dI.\end{aligned} \tag{13}$$

It can be obtained from (7) that

$$\lim_{I\to 0^+} P_f(\lambda|I) = 0 \tag{14}$$

and

$$\lim_{I\to +\infty} P_f(\lambda|I) = Q(-\sqrt{N}). \tag{15}$$

Hence, (13) becomes

$$\overline{P}_f(\lambda) = Q(-\sqrt{N}) \int_{\xi(\lambda)}^{+\infty} f(I) dI. \tag{16}$$

In the following, we prove that $\xi(\lambda) \approx \lambda$ for sufficiently large N. From (16), we define

$$\Phi(\lambda) = Q(-\sqrt{N}) \int_{\lambda}^{+\infty} f(I) dI. \tag{17}$$

The first derivations of $\Phi(\lambda)$ in (17) and $\overline{P}_f(\lambda)$ in (11) are, respectively, given by

$$\frac{\partial \Phi(\lambda)}{\partial \lambda} = -Q(-\sqrt{N}) f(\lambda) \tag{18}$$

and

$$\frac{\partial \overline{P}_f(\lambda)}{\partial \lambda} = -\int_0^{+\infty} \delta(I, \lambda) f(I) dI \tag{19}$$

with

$$\delta(I, \lambda) = \frac{1}{\sqrt{2\pi N^{-1}} I} e^{-\frac{(I-\lambda)^2}{2N^{-1}I^2}}. \tag{20}$$

It can be verified that $\delta(I, \lambda)$ approaches to the Dirac delta function centering at λ when N is sufficiently large. Hence, (19) gives

$$\frac{\partial \overline{P}_f(\lambda)}{\partial \lambda} \approx -f(\lambda). \tag{21}$$

Therefore, by comparing (18) and (21) with sufficiently large N, one can have

$$\frac{\partial \overline{P}_f(\lambda)}{\partial \lambda} \approx \frac{\partial \Phi(\lambda)}{\partial \lambda}. \tag{22}$$

As a result, we can approximate $\overline{P}_f(\lambda)$ to $\Phi(\lambda)$ under sufficiently large N, i.e.,

$$\overline{P}_f(\lambda) \approx \Phi(\lambda) = Q(-\sqrt{N}) \int_{\lambda}^{+\infty} f(I) dI. \tag{23}$$

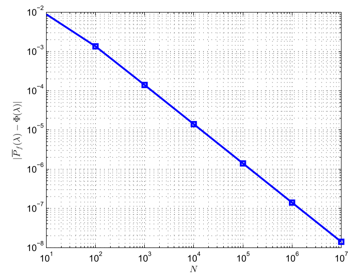

Fig. 2. Difference between $\overline{P}_f(\lambda)$ and $\Phi(\lambda)$ for different N.

Fig. 2 shows the difference between $\overline{P}_f(\lambda)$ and $\Phi(\lambda)$ for different N. It can be observed that the difference is as small as 0.01 even when $N = 10$. Thus, the solution to (12) is approximated to that to

$$Q(-\sqrt{N}) \int_{\lambda}^{\infty} f(I) dI = P_f^*. \tag{24}$$

By using

$$\int_{\lambda}^{\infty} f(I) dI = Q\left(\frac{\ln \lambda - \mu}{\sqrt{\nu}}\right), \tag{25}$$

an explicit expression for the decision threshold can be obtained as

$$\lambda \approx e^{\sqrt{\nu} Q^{-1}\left(\frac{P_f^*}{Q(-\sqrt{N})}\right)+\mu}. \tag{26}$$

B. SIR Wall

The average detection probability is given by

$$\begin{aligned}\overline{P}_d(\lambda) &= \mathbf{E}[P_d(\lambda|I)] \\ &= \int_0^{\infty} P_d(\lambda|I) f(I) dI\end{aligned} \tag{27}$$

where $P_d(\lambda|I)$ is given in (8). Substituting (26) into (27) gives

$$\overline{P}_d = \int_0^{\infty} Q\left(\frac{\lambda - (\sigma_s^2 + I)}{\sqrt{N^{-1}}(\sigma_s^2 + I)}\right) f(I) dI. \tag{28}$$

To achieve a target detection probability of P_d^*, one requires that

$$\int_0^{\infty} Q\left(\frac{\lambda - (\sigma_s^2 + I)}{\sqrt{N^{-1}}(\sigma_s^2 + I)}\right) f(I) dI \geq P_d^*. \tag{29}$$

Note that the left-hand side of (29) is an increasing function of the argument σ_s^2. Hence, it is easy to obtain the minimum value of σ_s^2 for (29), e.g., by the bisection algorithm. Let σ_s^{2*} be the minimum requirement of the signal power, and the SIR wall is given by

$$\text{SIR}_{\text{wall}} = \frac{\sigma_s^{2*}}{\sigma^2} \tag{30}$$

below which a reliable performance will no longer be achieved.

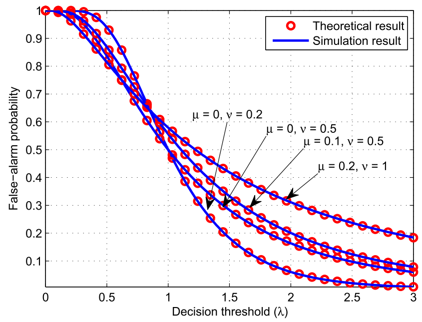

Fig. 3. False-alarm probability versus decision threshold when $N = 1000$.

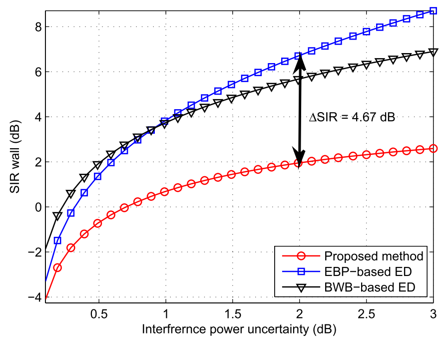

Fig. 4. SIR wall versus interference power uncertainty for $P_f^* \leq 0.1$ and $P_d^* \geq 0.9$ when $N = 1000$.

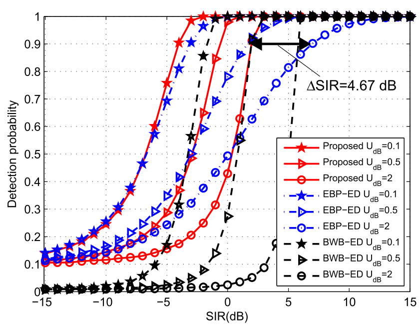

Fig. 5. Detection probabilities of the proposed method, BWB-based ED and EBP-based ED for different interference power uncertainties when $N = 1000$ and $P_f^* = 0.1$.

IV. Numerical Simulations

In this section, we verify the theoretical results and evaluate the detection performance of the proposed method through numerical simulations.

Fig. 3 shows the theoretical and Monte Carlo simulation results of the false-alarm probabilities versus decision threshold for different means and variances of the interference power when $N = 1000$. It can be observed that the theoretical results closely match with the Monte Carlo simulation results, which validates our theoretical analysis.

Fig. 4 shows SIR wall versus interference power uncertainty of the proposed method, the BWB-based ED in [3] and the EBP-based ED in [5] for $P_f^* \leq 0.1$ and $P_d^* \geq 0.9$ when $N = 1000$. In the simulations, the upper bound required in the BWB-based ED is selected so that the interference power will not exceed it with a probability of 99%. For the EBP-based ED, an unbiased interference power estimate with log-normal distribution is assumed. It can be seen from Fig. 4 that the proposed method has the lowest SIR wall than the others. At the interference power uncertainty of 2dB, the SIR wall of the proposed method is about 4.67 dB lower than that of EBP-based ED.

Fig. 5 shows the detection probabilities of the proposed method, BWB-based ED and EBP-based ED for different interference power uncertainties when $N = 1000$ and $P_f^* = 0.1$. It is observed that the proposed method has the best performance. To achieve the detection probability of $P_d^* = 0.9$, the proposed method has a performance improvement of over 4.67dB compared to the EBP-based ED, when the interference power uncertainty is 2dB.

V. Conclusion

In this letter, we have addressed the issue of unbounded support of interference power distribution in energy detection. An explicit expression for the decision threshold has been derived, and the SIR wall has been analyzed. Simulation results have verified the theoretical analyses and shown the superior performance of the proposed method.

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